On the Rate Loss of Multiple Description Source Codes and Additive Successive Refinement Codes

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Abstract — **The rate loss of a multi-resolution source code (MRSC) describes the difference between the rate needed to achieve distortion D_i in resolution i and the rate-distortion function $R(D_i)$. We generalize the rate loss definition and bound the rate losses of multiple description source codes (MDSCs) and additive MRSCs (AMRSCs). For a 2-description MDSC (2DSC), the rate loss of description i with distortion D_i is defined as $L_i = R_i - R(D_i)$, $i = 1, 2$, where R_i is the rate of the ith description; the rate loss associated with decoding the two descriptions together to achieve central distortion D_0 is measured as $L_0 = R_1 + R_2 - R(D_0)$ or as $L_{12} = L_1 + L_2$. We show that given an arbitrary source with variance σ^2, there exists a 2DSC with $L_1 \le 0.5$ and (a) $L_0 \le 1$ if $D_0 \le D_1 + D_2 - \sigma^2$, (b) $L_{12} \le 1$ if $1/D_0 \le 1/D_1 + 1/D_2 - 1/\sigma^2$, (c) $L_0 \le L_{G0} + 1.5$ and $L_{12} \le L_{G12} + 1$ otherwise, where L_{G0} and L_{G12} are the joint rate losses of a Normal$(0, \sigma^2)$ source. An AMRSC is an MRSC with the kth-resolution reconstruction equal to the sum of the first k side reproductions of an MDSC. We obtain one bound on the rate loss of an AMRSC.**

I. Introduction

The goal of multiple description source code (MDSC) design is to achieve a code that yields good rate-distortion performance under a variety of packet-loss scenarios. We measure 2DSC performance as $(R_1, R_2, D_0, D_1, D_2)$, where (R_i, D_i) $i \in \{1, 2\}$ are the expected rate and distortion for packet i and D_0 is the expected distortion in jointly decoding the two packets.

An additive multi-resolution source code (AMRSC) is an MDSC used as an MRSC [1]. A 2-stage AMRSC (A2RSC) encodes source X using two packets with expected rates R_1 and $\Delta R = R_2 - R_1$ respectively. The reproduction from packet 1 has expected distortion D_1 and the sum of the reproductions from both packets yields expected distortion $D_2 \le D_1$.

We measure the performance penalty of an MDSC or an AMRSC by the *rate loss* vector L. Given rate-distortion function $R(D)$, the rate loss of a 2DSC is $L_i = R_i - R(D_i)$ $(i = 0, 1, 2)$ (here $R_0 = R_1 + R_2$) and $L_{12} = L_1 + L_2$. The rate loss of an A2RSC is $L_i = R_i - R(D_i)$ $(i = 1, 2)$.

This paper focuses on finding source-independent rate loss bounds for MDSCs and AMRSCs. Bounds for more network source codes can be found in [3]. Rate loss bounds are useful for three reasons. (1) They describe the performance degradation associated with using the given code rather than the best single-resolution code with the same distortion. (2) They give elegant and often tight inner bounds on the region of achievable rates and distortions. For MDSC and AMRSC, these bounds are much simpler to analyze than existing alternatives. (3) Since the exact rate-distortion regions for AMRSCs and MDSCs are not known in general, the rate loss also gives a good bound on the distance between the best existing inner and outer bounds.

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II. Main Results

Let X be an i.i.d. source and assume the squared error distortion measure. Let σ^2 and $h(X)$ be the finite variance and finite differential entropy of X, respectively. Define $\mathcal{E} = 0.5\log(2\pi e\sigma^2) - h(X)$. We assume $0 < D_2 \le D_1 \le \sigma^2$.

A. Multiple Description Source Codes

Let $0 < D_0 < D_2$, $R_1^2 = (R_1, R_2)$, $D_0^2 = (D_0, D_1, D_2)$, and L_{G0} and L_{G12} denote L_0 and L_{12} for a Gaussian source. Define

$$\mathcal{R}(D_0^2) = \{R_1^2 : (R_1, R_2, D_0, D_1, D_2) \text{ is 2DSC-achievable}\}.$$

for each D_0^2. We partition the space of D_0^2 into 3 regions:

$$\begin{aligned} \mathcal{D}_1 &= \{D_0^2 : 0 \le D_0 \le D_1 + D_2 - \sigma^2\} \\ \mathcal{D}_3 &= \{D_0^2 : D_0 \ge (1/D_1 + 1/D_2 - 1/\sigma^2)^{-1}\}, \end{aligned}$$

and $\mathcal{D}_2 = \overline{\mathcal{D}_1 \cup \mathcal{D}_3}$. The following results give bounds on the values of $\mathcal{R}(D_0^2)$. By symmetry, each statement about L_1 can also be made to apply to L_2.

Theorem 1 *For any $D_0^2 \in \mathcal{D}_1$, there exists an $R_1^2 \in \mathcal{R}(D_0^2)$ with $L_1 \le 0.5$ and $L_0 \le 0.5\log[2(2\sigma^2 - D_0)/(\sigma^2 + D_0)] \le 1$.*

Theorem 2 *For any $D_0^2 \in \mathcal{D}_2$ with $D_1 < \sigma^2/2$, there exists an $R_1^2 \in \mathcal{R}(D_0^2)$ with $L_1 \le 0.5$ and $L_0 \le \min\{L_{G0} + 1.5, R(D_1) + 1\}$. The distance between this upper bound and a lower bound that can be derived from [2] for L_0 is less than $\min\{2\mathcal{E} + 0.5, \mathcal{E} + 1\}$. If the Shannon Lower Bound is tight at D_1, the distance is bounded by 2.*

Theorem 3 *For any $D_0^2 \in \mathcal{D}_2$ with $D_1 \ge \sigma^2/2$, there exists an $R_1^2 \in \mathcal{R}(D_0^2)$ with $L_1 \le 0.5$ and $L_0 \le 1$.*

Theorem 4 *For any $D_0^2 \in \mathcal{D}_2$, there exists an $R_1^2 \in \mathcal{R}(D_0^2)$ with $L_1 \le 0.5$ and $L_{12} \le 0.5\log(4D_2/D_0)$.*

An approach similar to that used in [2] leads to another bound for L_{12} in $\mathcal{D}_2$ and a constant bound in $\mathcal{D}_3$.

Theorem 5 *For any $D_0^2 \in \mathcal{D}_2$, there exists an $R_1^2 \in \mathcal{R}(D_0^2)$ with $L_1 \le 0.5$ and $L_{12} \le L_{G12} + 1$.*

Theorem 6 *For any $D_0^2 \in \mathcal{D}_3$, there exists an $R_1^2 \in \mathcal{R}(D_0^2)$ with $L_1 \le 0.5$ and $L_2 \le 0.5$.*

B. Additive Multi-Resolution Source Codes

Theorem 7 *For any $D_2 \le D_1$, there exists an A2RSC-achievable vector (R_1, R_2, D_1, D_2) with $L_1 \le 0.5$, and $L_2 \le \min\{0.5 + 0.5\log(D_1/D_2), 0.5 + 0.5\log(1 + (\sigma^2 - D_1)/D_2), \mathcal{E}\}$.*

References

[1] E. Tuncel and K. Rose, "Additive successive refinement", *Proceedings of ISIT 2001*, June, Washington, D.C..

[2] R. Zamir, "Gaussian codes and shannon bounds for multiple descriptions", *IEEE Trans.-IT*, 45:2629–2636, Nov. 1999.

[3] H. Feng and M. Effros, "On the rate loss of multiple description source codes and other network source codes", In preparation.